Highly Excited Friedmann Universe [a]

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Abstract: A highly excited Friedmann universe filled with a scalar field and radiation has been considered. On the basis of a direct solution to the quantum-mechanical problem with a well-defined time variable, it has been shown that such a universe can have features (energy density, scale factor, Hubble constant, density parameter, matter mass, equivalent number of baryons, age, dimensions of large-scale fluctuations, amplitude of fluctuations of cosmic microwave background radiation temperature) identical to those of the currently observed Universe.

1. Available cosmological data suggest that, from the point of view of quantum theory, the currently observed Universe is likely to be in a highly excited state [1]. This is confirmed by estimates of the number of the quantum state that corresponds to its averaged motion as a discrete unit [1-4]. In view of this, it is important to investigate cosmological systems featuring gravitational and matter fields and occurring in states with large quantum numbers by proceeding from a direct solution to the relevant quantum-mechanical problem.

A model of the Friedmann universe filled with a uniform scalar field has been proposed in [4]. This model, which is appropriate for constructing a quantum theory, features a well-defined time variable. The reference frame was specified there with the aid of a subsidiary matter source in the form of radiation (relativistic matter of any nature) that was assumed to be initially present in the cosmological system along with a scalar field that forms a nonzero cosmological constant in the early universe. The evolution of the universe filled with not only scalar field but also with radiation differs from that which is realized in the absence of radiation. The main difference lies here in the emergence of a new region that is accessible to a classical motion and which is bounded by the potential barrier existing in the system of scalar and gravitational fields. A quantum universe involving a slowly varying scalar field and occurring in low-lying (quasistationary) states has been analyzed in [4], where it has been shown that the dynamical model proposed there is compatible with the currently prevailing ideas of the early Universe. In this study, we will consider a quantum Friedmann universe with large quantum numbers characterizing the possible physical states of the gravitational and matter fields involved. On the basis of a solution to the quantum mechanical problem, it is shown that the universe in highly excited states can have features (energy density, scale factor, Hubble constant, density parameter, matter mass, equivalent number of baryons) identical to those in the currently observed Universe. The problem of the age of the Universe and the possible new mechanism that could generate fluctuations of the metric due to a finite width of quasistationary states and to the anisotropy of cosmic microwave background radiation are discussed in the Appendices.

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2. The wave function of the quantum Friedmann universe filled with radiation and a uniform scalar field ϕ specified by the potential $V(\phi)$ is determined by the Schrödinger equation [4]

$$2\, i\, \partial_T \Psi = \left(\partial_a^2 - \frac{2}{a^2}\, \partial_\phi^2 - U \right) \Psi, \tag{1}$$

where T is a privileged time coordinate related to the synchronous proper time t by the coordinate condition $dT/dt = 1/a$, a being the scale factor, while

$$U = a^2 - a^4\, V(\phi) \tag{2}$$

plays the role of the effective interaction potential in the cosmological system being considered. Here, we use the system of units in which $l = \sqrt{2G\hbar/3\pi c^3} = 1$ and $\tilde{\phi} = \sqrt{3c^4/8\pi G} = 1$.[1]

Possible solutions to equation (1) are determined by the properties of the scalar field involved. If $V(\phi)$ is everywhere positive definite, we can see that, in the variable a, the potential U has the form of a barrier with height $U_{max} = \frac{1}{4V}$ and width $\Delta a = a_2(E) - a_1(E)$, where $a_1(E)$ and $a_2(E)$, $a_1(E) < a_2(E)$, are the classical turning points determined from the condition $U = E$. With the aid of the equation of motion for the field ϕ, it can be shown that, if the field satisfies the condition $|\partial_t^2 \phi| \ll |\frac{dV}{d\phi}|$, the contribution of the operator $\frac{2}{a^2}\partial_\phi^2$ can be approximated by the term $-\frac{a^4}{18}\left(\frac{1}{H}\frac{dV}{d\phi}\right)^2$, where $H = \frac{\partial_t a}{a}$ is the Hubble constant; that is, this contribution is equivalent to the squared addition to the interaction U of the gradient of the potential $V(\phi)$. In this effective potential, stationary states cannot exist in the region $a \le a_1$. If, however, $V(\phi) \ll 1$, quasistationary states with lifetimes exceeding the Planck time can exist within the barrier [4]. The positions and widths of such states are determined by the solutions to equation (1) that satisfy the boundary condition in the form of a wave traveling toward greater values of a.

A general solution to equation (1) can be represented in the integral form

$$\Psi(a, \phi, T) = \int_0^\infty dE\, \mathrm{e}^{\frac{i}{2} ET}\, C(E)\, \psi_E(a, \phi), \tag{3}$$

where the function $C(E)$ characterizes the E distribution of the states of the universe at the instant $T = 0$, while $\psi_E(a, \phi)$ and E are, respectively, the eigenfunctions and the eigenvalues for the equation

$$\left(-\, \partial_a^2 + \frac{2}{a^2}\, \partial_\phi^2 + U - E \right) \psi_E = 0. \tag{4}$$

3. Let us now consider a quantum universe where $\left| \frac{1}{V}\frac{dV}{d\phi} \right|^2 \ll 1$. A solution to equation (4) can then be represented as

$$\psi_E(a, \phi) = \int_0^\infty d\epsilon\, \varphi_\epsilon(a, \phi)\, f_\epsilon(\phi; E), \tag{5}$$

[1] In order to go over to the system of units where $\hbar = c = 1$, we must use the relation $l\tilde{\phi}^2 = \sqrt{3/32\pi^3}\, m_p$ for the energy and the relation $(\tilde{\phi}/l)^2 = (9/16)\, m_p^4$ for the energy density, m_p being the Planck mass.

where φ_ϵ and ϵ are, respectively, the eigenfunctions and eigenvalues for the operator $[-\partial_a^2 + U]$ of the adiabatic approximation that correspond to continuum states at a fixed value of the field ϕ. The functions φ_ϵ can be normalized to the delta function $\delta(\epsilon - \epsilon')$. Their form greatly depends on the value of ϵ. For $\epsilon < U_{max}$, there are quasistationary states with $\epsilon = \tilde{\epsilon}_n \equiv \epsilon_n + i\,\Gamma_n$ in the system [4], and the main contribution to the integral in (5) over the interval $0 < \epsilon < U_{max}$ comes from the values $\epsilon \approx \epsilon_n$ and $a < R$, where $R \geq a_2(\epsilon)$. Here, the wave function has the form

$$\varphi_\epsilon = A(\epsilon)\,\varphi_\epsilon^{(0)}, \tag{6}$$

where the function $A(\epsilon)$ has a pole in the complex plane of ϵ at $\epsilon = \tilde{\epsilon_n}$, while $\varphi_\epsilon^{(0)}$ is the solution over the interval $0 < a < R$ that is regular at the point $a = 0$, normalized to unity, and weakly dependent on ϵ. Proceeding in a way similar to that adopted in the theory of quasistationary states for short-range potentials [5], we can show that the quantity $|A(\epsilon)|^2$ can be approximated by the delta function $\delta(\epsilon - \epsilon_n)$ in the case of the potential (2) as well. In this approximation, expression (5) can eventually be reduced to the expansion

$$\psi_E(a,\phi) = \sum_n \varphi_n(a,\phi)\, f_n(\phi;E) + \int_{U_{max}}^{\infty} d\epsilon\, \varphi_\epsilon(a,\phi)\, f_\epsilon(\phi;E), \tag{7}$$

where $\varphi_n = \varphi_{\epsilon_n}^{(0)}$ for $0 < a < R$ and $\varphi_n = 0$ for $a > R$ (a state of this type was considered in [6]), while $f_n(\phi;E) = \int_0^\infty da\, \varphi_n^*(a,\phi)\, \psi_E(a,\phi)$. In the limit of an impenetrable barrier, the function φ_n reduces to the wave function of a stationary state with a definite value of ϵ_n. In the case of $V \ll 1$ considered here, the contribution of the integral to the expansion in (7) can be disregarded, and the quantities $f_n(\phi;E)$ can be interpreted as the amplitudes of the probability that the universe is in the state $f_n(\phi;E)$ with a given value of the field ϕ. They satisfy the set of differential equations

$$\partial_\phi^2 f_n + \sum_{n'} K_{nn'}\,(\phi;E)\, f_{n'} = 0, \tag{8}$$

where

$$K_{nn'} = \langle\varphi_n|\partial_\phi^2|\varphi_{n'}\rangle + 2\,\langle\varphi_n|\partial_\phi|\varphi_{n'}\rangle\partial_\phi + \frac{1}{2}\,\langle\varphi_n|a^2|\varphi_{n'}\rangle\,(\epsilon_{n'} - E)\,. \tag{9}$$

Over the time interval $\Delta T < \frac{1}{\Gamma_n}$, we can disregard the possibility of decay and consider the quasistationary state as a stationary state that arises in place of the quasistationary state when the decay probability tends to zero. The wave function φ_n of such a stationary state and the corresponding eigenvalue ϵ_n can be found by perturbation theory by considering the interaction $a^4 V(\phi)$ as a small perturbation against a^2 (in the region $a < a_1$, we have $a^2 V < 1$). This yields

$$\begin{aligned}
\varphi_n \;&=\; |n\rangle - \frac{V}{4}\left[\frac{1}{8}\sqrt{N(N-1)(N-2)(N-3)}\,|n-2\rangle + \right.\\
&+\; \sqrt{N(N-1)}\left(N - \frac{1}{2}\right)|n-1\rangle - \\
&-\; \sqrt{(N+1)(N+2)}\left(N + \frac{3}{2}\right)|n+1\rangle - \\
&-\; \left.\frac{1}{8}\sqrt{(N+1)(N+2)(N+3)(N+4)}\,|n+2\rangle\right] + O\left(V^2\right),
\end{aligned} \tag{10}$$

$$\begin{aligned}\epsilon_n &= \epsilon_n^0 - \frac{3}{4} V\, [2N(N+1)+1] - \\ &- \frac{V^2}{4}\left[\frac{17}{2} N^3 + \frac{51}{4} N^2 + \frac{59}{4} N + \frac{21}{4}\right] + O\left(V^3\right), \end{aligned} \tag{11}$$

where $N = 2n+1$, while $\epsilon_n^0 = 2N+1$ and $|n\rangle$ are, respectively, an eigenvalue and the corresponding eigenfunction of the operator $[-\partial_a^2 + a^2]$. Quasistationary states are realized for $V < 0.08 = 4.5 \times 10^{-2} m_p^4$ and are characterized by values $\epsilon_n > 2.6$ and $\Gamma_n \ll 0.3$ [4]. The universe can undergo a tunnel transition to the region $a > a_2(\epsilon_n)$ from any quasistationary state.

In the early universe, the quantity $V(\phi)$ specifies the vacuum energy density, which determines the cosmological constant Λ at that era [7, 8]. In our Universe, the cosmological constant is very small. In the model featuring matter in the form of one scalar field, the reduction of the cosmological term can be described in terms of the potential $V(\phi(t))$, which decreases with time [3, 9]. A similar behavior of $V(\phi(t))$ is suggested by the results of investigations within inflationary models [8, 9]. It was shown in [4] that, with a nonzero probability, a quantum universe initially filled with radiation and a scalar field whose potential $V(\phi(t))$ decreases with time can evolve in the region bounded by the barrier. The expansion occurs here owing to transitions from lower to higher states via the interactions of the scalar and gravitational fields involved. In the zeroth approximation, the evolution of the universe can be described by considering transitions between unperturbed states $|n\rangle$ and $|n+1\rangle$ under the effect of the interaction $a^4 V$ [4]. In a more rigorous approach that takes into account variations in $V(\phi)$, the universe expands through transitions between the states φ_n and φ_{n+1} due to the gradient of the potential $V(\phi)$.

As the potential $V(\phi)$ decreases, the number of quantum states in which the universe can occur increases. As a result, the competition between the tunneling processes and transitions from one state to another arises in the universe that has not had time to undergo tunneling. Since the decay probability decreases exponentially with decreasing V, the probability that the universe is excited to states with large quantum numbers n in the course of time is nonzero. The probability that, in course of time, the universe in large-n states will occur in the region outside the barrier is negligibly small, because small values of V correspond to such states. In the limit $V \to 0$, the universe is completely locked in the region within the barrier with ϵ_n tending ϵ_n^0.

4. Let us investigate the properties of the universe in states with $n \gg 1$. In such states, $|V| \ll 1$ and $\Gamma_n \sim 0$, and we can use the wave function (10) in calculating the quantities $K_{nn'}$ in (9). As before, we assume that the potential $V(\phi)$ changes slowly in these states as well ($\left|\frac{dV}{d\phi}\right| \ll 1$). The derivatives $\partial_\phi \varphi_n$ in (9) can then be disregarded. Taking this into account and going over to the limit $n \gg 1$, we can then reduce equation (8) to the simplified form

$$\partial_\phi^2 f_n + \omega_n^2(\phi; E) f_n = 0, \tag{12}$$

where

$$\omega_n^2(\phi; E) = 2N^2\left[1 - \frac{E}{2N} - 2NV(\phi)\right]. \tag{13}$$

The potential of the field ϕ will be chosen in the form $V(\phi) = \frac{m^2}{2}\phi^2$. From the condition $|V| \ll 1$, it follows that the mass of the fields must be constrained by the condition

$m \ll \frac{m_p^2}{|\phi|}$. In this case, only the region $|\phi| \ll m_p$ corresponds to extremely large masses $m \geq m_p$. For masses satisfying the inequality $m \ll m_p$, the field ϕ can be determined over the entire interval $0 \leq |\phi| < \infty$; in this case, equation (12) reduces to the Schrödinger equation for a harmonic oscillator. Under such conditions, the field ϕ can oscillate near the minimum of the potential $V(\phi)$, causing the production of particles.[2] Substituting the explicit expression for $V(\phi)$ into (13) and introducing the new variable $x = (2m^2N^3)^{1/4}\phi$, we arrive at the equation

$$\partial_x^2 f_n + (z - x^2) f_n = 0, \tag{14}$$

where $z = \frac{\sqrt{2N}}{m}\left(1 - \frac{E}{2N}\right)$. Equation (14) has solutions at $z = 2s+1$, where $s = 0, 1, 2, \ldots$. Suppose that the universe occurs in a state with a definite quantum number s. The function f_n can then be represented as

$$f_{ns} = \frac{1}{\sqrt{s!}}\left(B_n^\dagger\right)^s f_{n0}, \quad B_n f_{n0} = 0, \quad f_{n0} = \left(\frac{1}{\pi}\right)^{1/4} \mathrm{e}^{-x^2/2}, \tag{15}$$

where f_{n0} is the state vector of the universe in the nth state at $s = 0$, while $B_n^\dagger = \frac{1}{\sqrt{2}}(x - \partial_x)$ and $B_n = \frac{1}{\sqrt{2}}(x + \partial_x)$ are the operators that, respectively, create and annihilate particles in this state and which satisfy the conventional commutation relations $[B_n, B_n^\dagger] = 1$ and $[B_n, B_n] = [B_n^\dagger, B_n^\dagger] = 0$. Since $n \gg 1$, small changes in n do not affect the physical state of the universe where there are s particles.

The condition of quantization of E has the form

$$E = 2N - (2N)^{1/2}(2s+1)m. \tag{16}$$

For $m \ll m_p$ and small s, the presence of the field ϕ has no effect on the properties of the universe because, in this case, the approximate equality $E \approx 2N$ holds to a high precision, so that the universe is dominated by radiation. A transition from the radiation-dominated universe to the universe where matter (in the form of particles produced by the field ϕ) prevails occurs when, owing to an increase in the number of particles, the second term in (16) becomes commensurate with the first one. Let us consider the case where there are many particles in a highly excited nth state of the universe. For $n \gg 1$ and $s \gg 1$, the wave function has the form

$$\psi_{ns}(a, \phi) = \varphi_n(a) f_{ns}(\phi), \tag{17}$$

where

$$\varphi_n(a) = \left(\frac{2}{N}\right)^{1/4} \cos\left(\sqrt{2N}\, a - \frac{N\pi}{2}\right), \tag{18}$$

$$f_{ns}(\phi) = \left(\frac{m(2N)^{3/2}}{4s}\right)^{1/4} \cos\left(\sqrt{2s}\,(2m^2N^3)^{1/4}\,\phi - \frac{s\pi}{2}\right). \tag{19}$$

[2] At $E = 0$, a similar mechanism leads to the production of particles by the inflaton field, which is identified with the scalar field ϕ [8]. In the case considered here, the field ϕ for $n \gg 1$ states can be treated as an effective field obtained upon averaging over the internal degrees of freedom of real physical fields.

This wave function corresponds to the semiclassical solution to equation (4) and is normalized to unity with allowance for the fact that the probability of finding the universe in the region $a > a_2$ is negligibly small.

Let us now calculate the energy density ρ_{tot} for matter and radiation in the universe described by the wave function (17). The energy density for a classical field ϕ and radiation is determined by the Einstein equation for the $\binom{0}{0}$ component and is given by [4, 10]

$$\rho = \frac{2}{a^6}\pi_\phi^2 + V + \frac{E}{a^4}, \tag{20}$$

where π_ϕ is the momentum canonically conjugate to the variable ϕ. Assuming that all the quantities in (20) are operator-valued, we set $\rho_{tot} = \langle\rho\rangle$, where averaging is performed with the wave function (17). Disregarding the variances $\langle a^2\rangle - \langle a\rangle^2$ and $\langle a^6\rangle - \langle a\rangle^6$, we obtain

$$\rho_{tot} = \frac{2}{\langle a\rangle^6}\langle\pi_\phi^2\rangle + \langle V\rangle + \frac{E}{\langle a\rangle^4}. \tag{21}$$

Calculating the expectation values in (21), we arrive at the total energy density in the universe in the form of the sum of the energy densities for matter and radiation as in the general theory of relativity [10]:

$$\rho_{tot} = \frac{193}{12}\frac{M_\phi}{\langle a\rangle^3} + \frac{E}{\langle a\rangle^4}. \tag{22}$$

Here, $M_\phi = ms$, and $\langle a\rangle = \sqrt{\frac{N}{2}}$ is the scale factor in the universe occurring in an $n \gg 1$ state. The constant E and the total mass M_ϕ are related by the equation

$$E = 4\langle a\rangle\left[\langle a\rangle - M_\phi\right]. \tag{23}$$

If

$$\langle a\rangle = M_\phi, \tag{24}$$

there is no radiation; we then have $\rho_{tot} = \rho_{sub}$ and

$$\langle a\rangle = \left(\frac{193}{12}\frac{1}{\rho_{sub}}\right)^{1/2}. \tag{25}$$

5. Suppose that the system being considered occurs in an $n \gg 1$, $s \gg 1$ state and that it is characterized by the scale-factor value of $\langle a\rangle \sim 10^{28}$cm. The condition in (24) will then be satisfied at $M_\phi \sim 10^{56}$g. This value coincides with the mass of matter in the observed part of our Universe [10]. On the other hand, we set $\frac{E}{\langle a\rangle^4}$ to the density of cosmic microwave background radiation energy in the present era, $\rho_\gamma^0 = 2.6 \times 10^{-10}$ GeV/cm^3 [11]. From (23), we then find that, for $\langle a\rangle = 10^{28}$cm, we have $\frac{M_\phi}{\langle a\rangle} = 1 - 0.7 \times 10^{-5}$; that is, the equality in (24) holds to a high precision in this case as well. We note that, although the absolute value of E is not small in the present era ($E \sim 10^{117}$ [4]), it is small in relation to $\langle a\rangle^2 \sim 10^{122}$ (all estimates are presented here in the system of units where $l = \tilde{\phi} = 1$) and can be disregarded. It is the ratio of these quantities that determines the accuracy to which relation (24) holds.

The current values of the scale factor ($\langle a\rangle \sim a_0 \sim 10^{28}$cm) and of the mean energy density ($\rho_{sub} \sim \rho_0 \sim 10^{-5}$ GeV/cm^3) satisfy relation (25). Associating the well-known equality $a_0 = \left(\frac{\Omega_0}{\Omega_0 - 1}\frac{1}{\rho_0}\right)^{1/2}$ with relation (25), we find that the effective value of the density parameter is $\Omega_0 = 1.066$; that is, the geometry of the universe with the above features is close to Euclidean geometry. It is characterized by the quantum-number values of $n \sim \langle a\rangle^2 \sim 10^{122}$ and $s \sim \frac{\langle a\rangle}{m}$. Taking the proton mass for m, we obtain $s \sim 10^{80}$. The above value of n complies with existing estimates for our Universe (see [1-4]), while s is equal to the equivalent number of baryons [10]. The value found for Ω_0 corresponds to the Hubble constant value of $H_0^{theory} \simeq 94$ km/(s Mpc). This value of Hubble constant lies within the limits of experimental uncertainty of the value of $H_0^{exp} = 80 \pm 17$ km/(s Mpc), which was obtained with the aid of the Hubble cosmic telescope [12].

The approach developed here makes it possible to obtain realistic estimates for the age of the Universe (see Appendix 1), for the proper dimensions of the nonhomogeneities of matter which are consistent with those of the observed large-scale structure of the Universe, and for the amplitude of the fluctuations of the cosmic microwave background radiation temperature. The resulting estimate for the last quantity is close to the value extracted from experimental data (see Appendix 2).

6. The above numerical estimates of the parameters of the universe are of an illustrative character. If we nevertheless associate them with our Universe, it can be concluded that the values observed in the present era for the scale factor, the mass of matter in the Universe, and the density of the cosmic microwave background radiation energy satisfy relation (23). The zero-order approximation, which corresponds to setting $\pi_\phi^2 = 0$ in (20), leads to a very small value of $\Omega_0 \simeq 0.1$, but it is very close to the lower boundary of the uncertainty interval for Ω_0 [11]. Although the potential $V(\phi)$ undergoes only small variations in response to changes in the field ϕ, the field ϕ itself changes fast, oscillating about the point $\phi = 0$, so that the approximation in which $\pi_\phi^2 = 0$ is invalid. The application of the present model in this approximation would result in the radiation-dominated universe; that is, it would not feature a mechanism capable of filling it with matter upon a slow descent of the potential $V(\phi)$ to the equilibrium position, which corresponds to the true vacuum. In states of the universe that are characterized by large values of the quantum numbers, the kinetic term of the scalar field ensures the density parameter value close to unity.

Replacement of the entire set of actually existing massive fields by some averaged massive scalar field seems physically justified for states of the universe that have large values of n. We can see that, by and large, such an averaged field describes correctly the global features of our Universe. It effectively includes visible baryon matter and dark matter, which is globally manifested on cosmological scales via gravitational interaction. The status of the field ϕ changes as we go over from one stage of universe evolution to another. In the early universe, the field ϕ ensures a nonzero value of the vacuum-energy density (cosmological constant) due to $V(\phi)$ values at which the equation for $\varphi_\epsilon(a, \phi)$ admits nontrivial solutions in the form of quasistationary states. In a later era, when the field ϕ descends to a minimum of the potential $V(\phi)$ and begins to oscillate about this minimum, it appears to be a source of some averaged matter filling the visible volume of the universe, which has linear dimensions on the order of $\sim \langle a\rangle$.

APPENDIX 1

The time $\frac{1}{H_0}$ corresponding to the theoretical Hubble constant value H_0^{theory} is equal

to 10^{10} yr, and the age of the Universe calculated by the standard formula $t = \frac{2}{3H}$ is $t_0 \approx 7 \times 10^9$ yr. This result is smaller than the expected value of $t_0 = (10 \div 20) \times 10^9$ yr [11], but it is close to that which corresponds to H_0^{exp}, $t_0 \approx 8 \times 10^9$ yr, highlighting the problem of the age of the Universe. It should be borne in mind, however, that the age of the universe is calculated by the formula $t = \frac{q}{H}$, where $q = \frac{1}{2}$ (for the equation of state $p = \frac{\rho}{3}$ or $\frac{2}{3}$ (for $p = 0$), which implies that $a = b\,t^q$, where the proportionality factor b is independent of t. In the model of the universe filled with matter [as represented by the field $\phi(t)$] and radiation, the factor b depends on the variable $\phi(t)$, which changes with time. The inclusion of this dependence leads to a greater value of t_0. By way of example, we indicate that, for $2\sqrt{V}t \ll 1$, the scale factor varies with t according to the law $a(t) \simeq (2\sqrt{\epsilon}\,t)^{1/2}$, where $\epsilon = \epsilon(\phi(t))$ [4]. From the above, it follows that, in order to calculate the age of the universe, we must consider the transcendental equation $t = [2H - \partial_t \ln \epsilon]^{-1}$. Since $\epsilon > 0$ and since it increases with time, we have $\epsilon > 0$.

In order to perform a numerical estimation, we assume that, in the time interval being considered, ϵ grows in proportion to a power of time: $\epsilon \sim t^\alpha$. The age of the universe is then given by the expression $t = \frac{1+\alpha}{2H}$. Since we have $\epsilon \sim 10^{117}$ and $t \sim 10^{61}$ (in l/c units), the exponent α is $\alpha \simeq 1.9$ [a similar power-law dependence follows from the relation $a^2 \sim n \sim \epsilon$ and from the classical expression for $a(t)$], whence we find that, if the Hubble constant set to its theoretical value H_0^{theory}, the age of the universe is $t_0 \simeq 15.2 \times 10^9$ yr. It should be noted that the above power-law dependence of ϵ on t ensures a correct relationship between the current value of the scale factor a_0 and the age t_0 of the universe: $a_0 \sim t_0 \sim 10^{61}$.

APPENDIX 2

A quasistationary state with a small, but finite value of the width Γ does not possess a definite value of ϵ. This uncertainty we denote it by $\delta\epsilon$ can serve as source of fluctuations of the metric. Let us demonstrate this explicitly. By associating $\epsilon + \delta\epsilon$ with the scale factor $a + \delta a$ and by using the solution to the Einstein equation in the region $a \leq a_1$ from [4], we find that the amplitude of fluctuations of the scale factor can be represented as

$$\frac{\delta a}{a} = \frac{1}{4}\,\frac{\delta\epsilon/\epsilon}{1 - \tanh\sqrt{V}t/2\sqrt{V\epsilon}}\,. \qquad (A.1)$$

Since $\delta\epsilon \lesssim \Gamma$, the fluctuations δa that were generated at the early stage of the evolution of the Universe will take the greatest values. In order to estimate them, we adopt the values of $V = 0.08$, $\epsilon = 2.6$, and $\delta\epsilon \lesssim 0.3$, corresponding to the time $t \sim 1$ [4]. We then have $\frac{\delta a}{a} \lesssim 0.04$. Since the dimension of large-scale fluctuations changed in direct proportion $a(t)$, this relation has remained valid up to the present time [13]. Taking this into account, we find that $\delta a \lesssim 130$ Mpc for the current value of $a \sim 10^{28}$ cm. On the order of magnitude, the above value corresponds to the scale of superclusters of galaxies [9]. Smaller values of $\delta\epsilon$ are peculiar to quantum states with smaller V. The fluctuations δa corresponding to them are smaller than those presented above and are expected to manifest themselves against the background of the large-scale structure. They can be associated with clusters of galaxies, galaxies themselves, and clusters of stars. Thus, the conclusions drawn on the basis of the model considered here are in line with the generally accepted concept that galaxies, their clusters, and other structures in the Universe are macroscopic manifestations of quantum fluctuations that have grown considerably [9].

Let us estimate the amplitude of fluctuations of the cosmic microwave background radiation temperature, $\delta T/T$. By using the relation $\epsilon = \rho_\gamma\, a^4$, where $\rho_\gamma = (\pi^2/15)\, T^4$ is the density of the cosmic microwave background radiation energy, we obtain

$$\frac{\delta T}{T} = \frac{1}{4}\frac{\delta\epsilon}{\epsilon} - \frac{\delta a}{a}\,. \qquad (A.2)$$

For $\sqrt{V}\,t \ll 1$, it follows from (A.1) and (A.2) that

$$\frac{\delta T}{T} \simeq \frac{t}{2\sqrt{\epsilon}}\,\frac{\delta a}{a}\,. \qquad (A.3)$$

For the time $t \sim 10^5$ yr ($t \sim 10^{56}$ in l/c units), which corresponds to the recombination of primary plasma (separation of radiation from matter), it can be found that, for the observed value of $\epsilon = 5.2 \times 10^{117}$ (in which case $\sqrt{V}\,t \leq \frac{t}{2\sqrt{\epsilon}} \sim 0.7 \times 10^{-3}$), the sought amplitude of fluctuations of cosmic microwave background temperature radiation at $\frac{\delta a}{a} \lesssim 0.04$ can be estimated as

$$\frac{\delta T}{T} \lesssim 2.8 \times 10^{-5}. \qquad (A.4)$$

Upon recombination, the fluctuations of the temperature undergo no changes; therefore, measurement of the quantity $\delta T/T$ for the present era furnishes information about the Universe at the instant of last interaction of radiation with matter. The estimate in (A.4) is in good agreement with experimental data on cosmic microwave background radiation (see [14]).

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